4-2 Activity Draft

1-1-1 Estimation with polynomials (scientific notation problem).

**4-2-1 Parabolas**

Show that, in general, we need three points on a parabola to generate it’s function formula. However, if we are given the vertex of the parabola, we need only one other point to find the formula (Perhaps put with systems?).

S-Z: p. 196 Example 2.3.3: Recall that the profit (defined on page 82) for a product is defined by the equation Profit = Revenue − Cost, or P(x) = R(x) − C(x). In Example 2.1.7 the weekly revenue, in dollars, made by selling x PortaBoy Game Systems was found to be R(x) = −1.5x 2 + 250x with the restriction (carried over from the price-demand function) that 0 ≤ x ≤ 166. The cost, in dollars, to produce x PortaBoy Game Systems is given in Example 2.1.5 as C(x) = 80x + 150 for x ≥ 0. 1. Determine the weekly profit function P(x). 2. Graph y = P(x). Include the x- and y-intercepts as well as the vertex and axis of symmetry. 3. Interpret the zeros of P. 4. Interpret the vertex of the graph of y = P(x). 5. Recall that the weekly price-demand equation for PortaBoys is p(x) = −1.5x + 250, where p(x) is the price per PortaBoy, in dollars, and x is the weekly sales. What should the price per system be in order to maximize profit?

S-Z: Example 2.3.4 p. 197: Much to Donnie’s surprise and delight, he inherits a large parcel of land in Ashtabula County from one of his (e)strange(d) relatives. The time is finally right for him to pursue his dream of farming alpaca. He wishes to build a rectangular pasture, and estimates that he has enough money for 200 linear feet of fencing material. If he makes the pasture adjacent to a stream (so no fencing is required on that side), what are the dimensions of the pasture which maximize the area? What is the maximum area? If an average alpaca needs 25 square feet of grazing area, how many alpaca can Donnie keep in his pasture?

S-Z: p. 201: #21: What is the largest rectangular area one can enclose with 14 inches of string?

S-Z: p. 201: #17: The temperature T, in degrees Fahrenheit, t hours after 6 AM is given by: T(t) = − 1 2 t 2 + 8t + 32, 0 ≤ t ≤ 12 What is the warmest temperature of the day? When does this happen?

APC: p. 52: Activity 1.5.2: Open a browser and point it to Desmos. In Desmos, enter q(x) = ax^2 + bx + c; you will be prompted to add sliders for a, b, and c. Do so. Then begin exploring with the sliders and respond to the following questions. a. Describe how changing the value of a impacts the graph of q. b. Describe how changing the value of b impacts the graph of q. c. Describe how changing the value of c impacts the graph of q. d. Which parameter seems to have the simplest effect? Which parameter seems to have the most complicated effect? Why? e. Is it possible to find a formula for a quadratic function that passes through the points (0, 8), (1, 12), (2, 12)? If yes, do so; if not, explain why not.

APC: p. 55: Activity 1.5.3: . Reason algebraically using appropriate properties of quadratic functions to answer the following questions. Use Desmos to check your results graphically. a. How many quadratic functions have x-intercepts at (−5, 0) and (10, 0) and a yintercept at (0, −1)? Can you determine an exact formula for such a function? If yes, do so. If not, explain why. b. Suppose that a quadratic function has vertex (−3, −4) and opens upward. How many x-intercepts can you guarantee the function has? Why? c. In addition to the information in (b), suppose you know that q(−1) −3. Can you determine an exact formula for q? If yes, do so. If not, explain why. d. Does the quadratic function p(x) −3(x + 1) 2 + 9 have 0, 1, or 2 x-intercepts? Reason algebraically to determine the exact values of any such intercepts or explain why none exist. e. Does the quadratic function w(x) −2x 2 + 10x − 20 have 0, 1, or 2 x-intercepts? Reason algebraically to determine the exact values of any such intercepts or explain why none exist.

Calc-Medic 2.1: . 2. Write an equation for the parabola, in vertex form, with vertex (5,12) that passes through the point (7,20).

3. a. Write an equation for the parabola shown below in vertex form, factored form, and standard form. b. What is the advantage of writing the quadratic in each of the different forms?

MFG: 6.1: Checkpoint 6.5: . How can you use a graph to factor a quadratic expression?

MFG 6.1: Checkpoint 6.8: Solve by factoring: (t−3)2=3(9−t)

Solutions: t= [Separate different values with a comma.]

MFG: 6.1: Checkpoint 6.10: What happens to the x-intercepts when you multiply the right side of y=ax2+bx+c by 3? (Perhaps for 7-1-1?)

They are tripled.  
 They are divided by 3.  
 They move 3 units to the right.  
 They are unchanged.

MFG 6.1 Checkpoint 6.12: Explain why the solutions of (x−3)(x−6)=10(x−3)(x−6)=10 are not 3 and 6. (Perhaps for 6-2-1?)

MFG 6.1: 6.18: Which statement is true?

All rectangles with the same perimeter have the same area.  
 The solutions of x(18−x)=80 are 18 and 80.  
 If the perimeter of a rectangle is 20 cm, the largest area it can have is 20 sq cm.  
 If you know the x-intercepts of the graph of y=x2+bx+c, you can write it in factored form.

1. MFG 6.3: 6.58: Find the x-intercepts and the vertex of the parabola y=6x−x2.

x-intercepts: Note: Use a comma to separate different points.

Vertex:

1. Verify your answers by graphing the function in the window.

MFG: 6.3 HW #8:

Match each function with its graph. In each equation, b>0.b>0.

1. y=−bxy=−bx
2. y=−bx2y=−bx2
3. y=b−x2y=b−x2
4. y=x−by=x−b
5. y=b−xy=b−x
6. y=x2−bxy=x2−bx

MFG 6.4 6.71: Late Nite Blues finds that it can sell 600−15x600−15x pairs of jeans per week if it charges xx dollars per pair. (Notice that as the price increases, the number of pairs of jeans sold decreases.)

1. Write an equation for the revenue as a function of the price of a pair of jeans.
2. Graph the function.
3. How much should Late Nite Blues charge for a pair of jeans in order to maximize its revenue?

MFG 6.4: HW #13: A travel agent offers a group rate of $2400$2400 per person for a week in London if 1616 people sign up for the tour. For each additional person who signs up, the price per person is reduced by $100.$100.

1. Let xx represent the number of additional people who sign up. Write expressions for the total number of people signed up, the price per person, and the total revenue.
2. How many people must sign up for the tour in order for the travel agent to maximize her revenue?

(Note: In previous problems in 6.4, do the same type, but do numerical examples before generalizing).

**4-2-2 Definition of Polynomials**

Give relationship to place value: Quickly determine f(10) with 5th degree poly with single-digit coefficients and adding (missing x^2 term).. Perhaps also Taylor Series?

Examine plugging in x=100 into 4th degree polynomial. Analogy of winning $1000 vs. $1 million dollars in the lottery (Both big, but one dominates the other).

S-Z p. 249 #38. Here are a few other questions for you to discuss with your classmates. (a) How many local extrema could a polynomial of degree n have? How few local extrema can it have? (b) Could a polynomial have two local maxima but no local minima? (c) If a polynomial has two local maxima and two local minima, can it be of odd degree? Can it be of even degree? (d) Can a polynomial have local extrema without having any real zeros? (e) Why must every polynomial of odd degree have at least one real zero? (f) Can a polynomial have two distinct real zeros and no local extrema? (g) Can an x-intercept yield a local extrema? Can it yield an absolute extrema? (h) If the y-intercept yields an absolute minimum, what can we say about the degree of the polynomial and the sign of the leading coefficient?

4-2-3 Shape of polynomials

Take a linear polynomial and keep multiplying by linear factors (What happens to what we had with zeros, end behavior, etc.?).

FM; 5.3: #10: Given the graph of g(x) on the right, identify the following: a. Local minimum value(s) b. Local maximum value(s) c. Minimum Degree d. Write out a possible function. Leave it in factored form.

FM 5.3 #3-4 (Perhaps for later): ܲሺݔሻ is cubic. ܲሺ0ሻ ൌ 0, ܲሺ2ሻ ൌ െ4, and ܲሺݔሻ ൐ 0 only when ݔ൐ 4. Sketch a possible graph, then find the equation of ܲሺݔሻ. (Your equation may be left in factored form, but don’t forget to find the leading coefficient.) 4. Write and graph the equation for a quartic function passing through the point ሺ3, െ4ሻ and tangent to the ݔ‐axis at ݔൌ5 and ݔ ൌ െ2. Don’t forget to find the leading coefficient.

Active Reading 9.2.7: Let p(x)=(x−2)(x+1)(2x−6). Thinking about the linear factors of p as lines, each with its own root, choose which option below could be the graph of p(x). Gives 4 graphs.

Active Reading: 9.2.14: The graph below shows: f(x)=x(x+3)(x−4)

Use the checkbox to graph the power function y=x3.

Now use the slider to zoom out, and observe the shape of the graph.

After zooming out, how do the graphs of y=x(x+3)(x−4) and y=x3 compare to each other?

Answer: The graphs are

?

very different

very similar

Complete the following statment:

The

?

end behavior

of the function x(x+3)(x−4) is like

?

x^2

x^3

Now, use algebra to expand the formula x(x+3)(x−4). What is the leading term (the one with the highest power on x)?

Active Reading 9.2.23: Our guess for the formula, f(x)=(x−1)(x−3)(x−4), actually matches one of the graphs. Use the input value x=2 to decide which of the graphs matches our formula.: 3 graphs given.

Active Reading 9.3: #14: Graph each of the polynomials listed below. Make sure your graph shows all intercepts and exhibits proper end behavior. Then enter the letter corresponding to the correct graph next to each formula for P(x).

**1.** P(x)=13x3(x+2)(x−3)2  
 **2.** P(x)=−x3+2x2+x−2  
 **3.** P(x)=(x2−2x−3)2  
 **4.** P(x)=−14(x−1)3(x+3)  
 **5.** P(x)=2x3+x2−x

Active Reading: 9.3 #15HW: Find a formula for the polynomial of least degree that is graphed below. The polynomial has x-intercepts −2,−1,1 and y-intercept 66.

APC: p. 265 Activity 5.2.2: By experimenting with coefficients in Desmos, find a formula for a polynomial function that has the stated properties, or explain why no such polynomial exists. (If you enter p(x)=a+bx+cx^2+dx^3+fx^4+gx^5 in Desmos², you’ll get prompted to add sliders that make it easy to explore a degree 5 polynomial.) a. A polynomial p of degree 5 with exactly 3 real zeros, 4 turning points, and such that limx→−∞ p(x) +∞ and limx→∞ p(x) −∞. b. A polynomial p of degree 4 with exactly 4 real zeros, 3 turning points, and such that limx→−∞ p(x) +∞ and limx→∞ p(x) −∞. c. A polynomial p of degree 6 with exactly 2 real zeros, 3 turning points, and such that limx→−∞ p(x) −∞ and limx→∞ p(x) −∞. d. A polynomial p of degree 5 with exactly 5 real zeros, 3 turning points, and such that limx→−∞ p(x) +∞ and limx→∞ p(x) −∞.

Calc Medic: 2.3: #2: Graph ݃(ݔ) = (ݔ − 2)(ݔ − 2)(ݔ + 2) on the coordinate plane and identify the following: a. X-intercept(s): b. Y-intercept: c. How many times does the curve change directions? d. What is different about the behavior of the graph at ݔ = 2 and at ݔ− = 2? Why do you think this happens?

Calc-Medic: 2.3; Given the polynomial to the right, which of the following could be the factored form of the function: (4 + ݔ)(1 − ݔ)(3 − ݔ) = (ݔ)݂ .a (4 − ݔ)(1 + ݔ)(3 + ݔ) = (ݔ)݂ .b = (ݔ)݂ .c ଵ ଺ (4 + ݔ)(1 − ݔ)(3 − ݔ) − = (ݔ)݂ .d ଵ ଺ (

Calc-Medic 2.4: Consider ݃ሺݔሻ = 3ݔ ݔ + ସ ଷ + 5 a. What happens to the y-values as the x-values get bigger and bigger (at the right end of the graph)? Try a few values to investigate. b. What happens to the y-values as the x-values get more and more negative (at the left end of the graph)? Try a few values to investigate.

Calc-Medic 2.4: Check Your Understanding 1. Describe the end behavior of the graph to the right. 2. Is it possible for this graph to have a degree of 5? Why or why not? 3. Which of the following terms, when added to the given polynomial, will change the end behavior? Check all that apply. ݔ2 = −ݕ ݔ5଻ + ଺ − 24 ݔ− ¨ ଼ ¨ −3ݔ ହ ¨ 5ݔ ଻ ¨ 1000 ¨ −300

Calc-Medic: 2.4: Match the polynomial to the graph without using a calculator or Desmos. ݔ2 = −ݕ .1 ଷ + 3ݔ + 1 2. ݕ= ଵ ଷ ݔ ݔ − ଷ ଶ − ସ ଷ ݔ3 = ݕ .3 ଶ + 2 4. ݕ− = ݔ ସ + 3ݔ ଶ + 3

MFG: 7.1: 7.7: The degree of a product of polynomials is the same as

A) the product of the degrees of the factors.  
 B) the number of terms in the product.  
 C) the sum of the degrees of the factors.  
 D) the sum of the lead coefficients.

MFG: 7.1: 7.10: Explain the difference between the expressions (a+b)3(a+b)3 and a3+b3.

MFG: 7.1: 7.12: Which statement is true?

A cubic polynomial y=ax3+bx2+cx+d describes y as a function of x.  
 We cannot factor the sum of two cubes.  
 (2x+3y)3=8x3+27y3  
 A cubic polynomial must have 4 terms

MFG: 7.1: 7.15: A closed box has a square base of length and width xx inches and a height of 88 inches, as shown at right.

1. Write a polynomial function S(x)S(x) that gives the surface area of the box in terms of the dimensions of the base.

MFG: 7.1 HW #48: A Norman window is shaped like a rectangle whose length is twice its width, with a semicircle at the top (see the figure). Write a polynomial, A(x),A(x), that gives its area.

1. MFG: 7.1 HW #49: A grain silo is built in the shape of a cylinder with a hemisphere on top (see the figure). Write an expression for the volume of the silo in terms of the radius and height of the cylindrical portion of the silo.
2. If the total height of the silo is five times its radius, write a polynomial function V(r)V(r) in one variable for its volume.

1. MFG 7.1: HW #50: A cold medication capsule is shaped like a cylinder with a hemispherical cap on each end (see the figure). Write an expression for the volume of the capsule in terms of the radius and length of the cylindrical portion.
2. If the radius of the capsule is one-fourth of its overall length, write a polynomial function V(r)V(r) in one variable for its volume.

MFG: 7.1: HW#56: A soup bowl has the shape of a hemisphere of radius 66 centimeters. The volume of the soup in the bowl, V=f(x),V=f(x), is a function of the depth, x,x, of the soup.

1. What is the domain of f?f? Why?
2. The function ff is given by

f(x)=6πx2−π3x3f(x)=6πx2−π3x3

Graph the function on its domain.

1. What is the volume of the soup if it is 33 centimeters deep?
2. What is the maximum volume of soup that the bowl can hold?
3. Find the depth of the soup (to within 22 decimal places of accuracy) when the bowl is filled to half its capacity.

MFG: 7.1: HW#55 A doctor who is treating a heart patient wants to prescribe medication to lower the patient's blood pressure. The body's reaction to this medication is a function of the dose administered. If the patient takes xx milliliters of the medication, his blood pressure should decrease by R=f(x)R=f(x) points, where f(x)=3x2−13x3f(x)=3x2−13x3

1. For what values of xx is R=0?R=0?
2. Find a suitable domain for the function and explain why you chose this domain.
3. Graph the function ff on its domain.
4. How much should the patient's blood pressure drop if he takes 22 milliliters of medication?
5. What is the maximum drop in blood pressure that can be achieved with this medication?
6. There may be risks associated with a large change in blood pressure. How many milliliters of the medication should be administered to produce half the maximum possible drop in blood pressure?

MFG 7.1: HW#59 The total annual cost of educating postgraduate research students at an Australian university, in thousands of dollars, is given by the function

C(x)=0.0173x3−0.647x2+9.587x+195.366C(x)=0.0173x3−0.647x2+9.587x+195.366

where xx is the number of students, in hundreds. (Source: Creedy, Johnson, and Valenzuela, 2002)

1. Graph the function in a suitable window for up to 35003500 students.
2. Describe the concavity of the graph. For what value of xx is the cost growing at the slowest rate?
3. Approximately how many students can be educated for $350,000?